Solutions to Problem 1.

a. Note that $\Lambda(22) - \Lambda(18) = 87 - 74 = 13$, so $Z_{22} - Z_{18}$ is a Poisson random variable with parameter 13. Therefore,

$$\Pr\{Z_{22} - Z_{18} \leq 12\} = \sum_{j=0}^{12} \frac{e^{-13}(13)^j}{j!} \approx 0.4631$$

b. Note that $\Lambda(24) - \Lambda(12) = 90 - 44 = 46$, so $Z_{24} - Z_{12}$ is a Poisson random variable with parameter 46. Therefore,

$$\begin{aligned}\Pr\{Z_{24} \geq 80 \mid Z_{12} = 40\} &= \Pr\{Z_{24} - Z_{12} \geq 40 \mid Z_{12} = 40\} \\ &= \Pr\{Z_{24} - Z_{12} \geq 40\} \\ &= 1 - \Pr\{Z_{24} - Z_{12} \leq 39\} \\ &1 - \sum_{j=0}^{39} \frac{e^{-46}(46)^j}{j!} \approx 0.8307\end{aligned}$$

c. $\Lambda(24) = 90$ is the expected number of phone calls to the dispatch over the course of the entire day.